

# Picture This

## Increasing Math and Science Learning by Improving Spatial Thinking



BY NORA S. NEWCOMBE

**A**lbert Einstein's scientific accomplishments so impressed the world that his name is shorthand for intelligence, insight, and creativity. To be an Einstein is to be inconceivably brilliant, especially in math and science. Yet Albert Einstein was famously late to talk, and he described his thinking processes as primarily nonverbal. "The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought," he once said. "[There are] more or less clear images."<sup>1</sup> Research on his brain, preserved after death, has seemed to support his claim of thinking in spatial images: Sandra Witelson, a neuroscientist in Canada,

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found that his parietal cortex, an area of the brain used for spatial and mathematical thinking, was unusually large and oddly configured,<sup>2</sup> and likely supported him in imagining the universe in innovative ways.

Einstein was unique, but he certainly was not the only scientist to depend on his ability to think spatially. Watson and Crick's discovery of the structure of DNA, for example, was centrally about fitting a three-dimensional spatial model to existing flat images of the molecule. The fact is, many people who work in the sciences rely on their ability to think spatially, even if they do not make grand discoveries. Geoscientists visualize the processes that affect the formation of the earth. Engineers anticipate how various forces may affect the design of a structure. And neurosurgeons draw on MRIs to visualize particular brain areas that may determine the outcome of a surgical procedure.

So, is spatial thinking really a key to science, technology, engineering, and mathematics—the so-called STEM disciplines? Yes. Scores of high-quality studies conducted over the past 50 years indicate that spatial thinking is central to STEM success. One of the most important studies is called Project Talent; it followed

approximately 400,000 people from their high school years in the late 1950s to today.<sup>3</sup> It found that people who had high scores on spatial tests in high school were much more likely to major in STEM disciplines and go into STEM careers than those with lower scores, even after accounting for the fact that they tended to have higher verbal and mathematical scores as well. Similar results have been found in other longitudinal studies: one began in the 1970s and tracked the careers of a sample of gifted students first studied in middle school;<sup>4</sup> another began in the 1980s with observing the block play of preschoolers and followed their mathematics learning through high school.<sup>5</sup>

In short, the relation between spatial thinking and STEM is a robust one, emerging for ordinary students and for gifted students, for men and for women, and for people who grew up during different historical periods. Spatial thinkers are likely to be more

interested in science and math than less spatial thinkers, and are more likely to be good enough at STEM research to get advanced degrees.

So, would early attention to developing children’s spatial thinking increase their achievement in math and science, and even nudge them toward STEM careers? Recent research on teaching spatial thinking suggests the answer may be yes.

### What Do We Mean by Spatial Thinking?

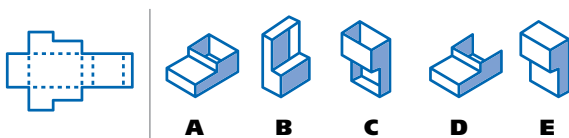
So far, we have been casual in using the term “spatial thinking.” But what do we really mean by it? Spatial thinking concerns the locations of objects, their shapes, their relations to each other, and the paths they take as they move. All of us think spatially in many everyday situations: when we consider rearranging the furniture in a room, when we assemble a bookcase using a diagram, or

## Tests of Spatial Thinking

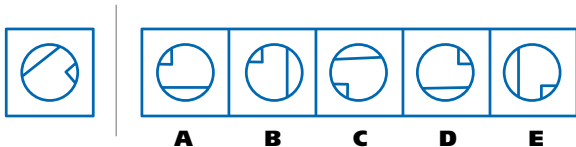
The following four tests were used in the Project Talent study. Here, each is briefly described and a sample item is provided. Answers for the sample items are on page 43 after the endnotes.

—EDITORS

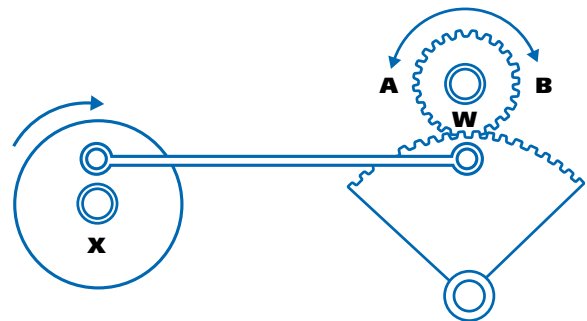
**1. Three-dimensional spatial visualization:** Each problem in this test has a drawing of a flat piece of metal at the left. At the right are shown five objects, only one of which might be made by folding the flat piece of metal along the dotted lines. You are to pick out the one of these five objects which shows just how the piece of flat metal will look when it is folded at the dotted lines. When it is folded, no piece of metal overlaps any other piece or is enclosed inside the object.



**2. Two-dimensional spatial visualization:** In this test each problem has one drawing at the left and five similar drawings to the right of it, but only one of the five drawings on the right exactly matches the drawing at the left if you turn it around. The rest of the drawings are backward even when they are turned around. For each problem in this test, choose the one drawing which, when turned around or rotated, is exactly like the basic drawing at the left.

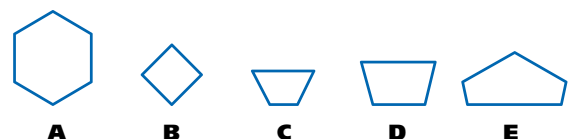
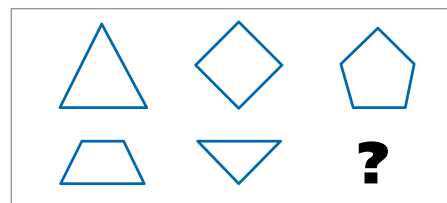


**3. Mechanical reasoning:** This is a test of your ability to understand mechanical ideas. You will have some diagrams or pictures with questions about them. For each problem, read the question, study the picture above it, and mark the letter of the answer on your answer sheet.



While wheel **X** turns round and round in the direction shown, wheel **W** turns  
 A. in direction **A**.  
 B. in direction **B**.  
 C. first in one direction and then in the other.

**4. Abstract reasoning:** Each item in this test consists of a set of figures arranged in a pattern, formed according to certain rules. In each problem you are to decide what figure belongs where the question mark is in the pattern.... The items have different kinds of patterns and different rules by which the drawings change.



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when we relate a map to the road ahead of us. We also use spatial thinking to describe nonspatial situations, such as when we talk about being *close to* a goal or describe someone as an *insider*.

This general description is helpful, but in conducting research, precise definitions are necessary. For the Project Talent study, spatial thinking was defined by the four tests used to assess it; a sample item from each of those four tests is shown in the box on page 30.<sup>6</sup> The first test asks us to imagine folding a two-dimensional shape into a three-dimensional one. The second asks us to mentally rotate a two-dimensional shape. The third asks us to imagine mechanical motion. The fourth asks us to see spatial patterns and progressions.

Tests like these four have been around for a century or so, and they remain useful assessments of spatial ability. But they do not cover the full range of abilities that fall under the term “spatial thinking,” so today’s researchers are working on developing new assessments. For example, one very different kind of spatial thinking involves navigating around the wider world. Many people think that, to get where we are heading, we need to be able to form a mental map of the environment.<sup>7</sup> It appears that some of us are much better than others at forming these integrated representations.<sup>8</sup> Spatial thinking of this kind may also be relevant to STEM success, but this idea has not yet been tested, largely because we lack good tests of navigation ability that can be given to large samples of students. Computer technology may soon allow such assessments.

To really understand what spatial thinking is, we must be clear about what it is not. First, spatial thinking is not a substitute for verbal or mathematical thinking. Those who succeed in STEM careers tend to be very good at all three kinds of thinking. Second, given the popularity of the notion that students have learning styles—i.e., that they are visual, auditory, or kinesthetic learners—it’s important to understand that spatial thinking is not a learning style. The truth is that there is virtually no support for learning styles in the research literature. While students may have preferences, all of us (with very rare exceptions) learn by seeing, hearing, and doing.\* Likewise, all of us (with very rare exceptions) think verbally, mathematically, and spatially. So teachers should be trying to provide students with the content knowledge, experiences, and skills that support development of all three ways of thinking.

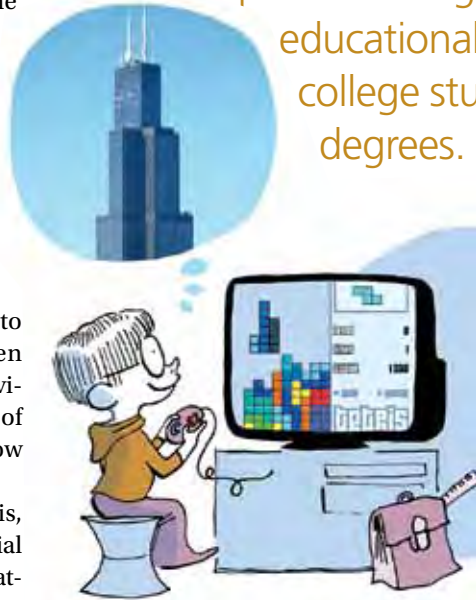
### Can Spatial Thinking Actually Be Improved?

Since spatial thinking is associated with skill and interest in STEM fields (as well as in other areas, such as art, graphic design, and architecture), the immediate question is whether it can be

improved. Can we educate children in a way that would maximize their potential in this domain? Americans often believe that their abilities are fixed, perhaps even at birth;<sup>9</sup> it is not uncommon to hear that a person was born with a gift for mathematics or a difficulty in learning foreign languages. But there is mounting evidence that this is not the case.<sup>10</sup> Abilities grow when students, their parents, and their teachers believe that achievement follows consistent hard work and when anxiety about certain areas, such as math, is kept low.<sup>†</sup>

What about spatial thinking in particular—is it malleable? Definitely. We have known for some time that elementary school

Spatial training has been found to improve educational outcomes, such as helping college students complete engineering degrees.



children’s spatial thinking improves more over the school year than over the summer months.<sup>11</sup> A recent meta-analysis (which integrated the results of all the high-quality studies of spatial malleability conducted over the past few decades) showed substantial improvements in spatial skill from a wide variety of interventions, including academic coursework, task-specific practice, and playing computer games that require spatial thinking, such as Tetris (a game in which play-

ers rotate shapes to fit them together as they drop down the screen).<sup>12</sup> Furthermore, these improvements were durable, and transferred to other tasks and settings. For example, when undergraduates were given extended, semester-long practice on mental rotation, through taking the test repeatedly and also through weekly play of Tetris, training effects were massive in size, lasted several months, and generalized to other spatial tasks such as constructing three-dimensional images from two-dimensional displays.<sup>13</sup> Along similar lines, undergraduates who practiced either mental rotation or paper folding daily, for three weeks, showed transfer of practice gains to novel test items, as well as transfer to the other spatial tasks they had not practiced.<sup>14</sup> Spatial training has also been found to improve educational outcomes, such as helping college students complete engineering degrees.<sup>15</sup>

While many studies have found that spatial thinking can be improved, researchers have found some important differences between high- and low-ability participants. For low-ability participants, there is an initial hump to get over. They improve slowly,

\*Instead of tailoring lessons to students’ supposed learning styles, teachers should be concerned with tailoring their lessons to the content (e.g., showing pictures when studying art and reading aloud when studying poetry). For a thorough explanation of this, see “Do Visual, Auditory, and Kinesthetic Learners Need Visual, Auditory, and Kinesthetic Instruction?” by Daniel T. Willingham in the Summer 2005 issue of *American Educator*, available at [www.aft.org/newspubs/periodicals/ae/issues.cfm](http://www.aft.org/newspubs/periodicals/ae/issues.cfm).

†Summing up 30 years of research, Daniel T. Willingham wrote, “Intelligence can be changed through sustained hard work.” For his explanation of the genetic and environmental influences on intelligence, see the sidebar on page 10 of the Spring 2009 issue of *American Educator*, available at [www.aft.org/newspubs/periodicals/ae/issues.cfm](http://www.aft.org/newspubs/periodicals/ae/issues.cfm).

if at all, for the first half-dozen or so sessions.\* But if they persevere, faster improvement comes, so it's important that students (and teachers) not give up.<sup>16</sup> High-ability participants do not have an initial hump, but they still can improve. Even people who are spatially proficient turn out to be not nearly as proficient as they could be, and they can attain even higher levels of excellence through fun activities like playing Tetris.<sup>17</sup> While playing Tetris may not fit into the school day, it might be offered in afterschool settings or be suggested to students as a weekend or summer activity (in moderation, of course). (Other spatial thinking activities that fit better into academic studies, such as why the earth has seasons, are discussed later.)

In addition to practicing spatial thinking tasks like those shown in the box on page 30, well-conceived symbolic representations, analogies, and gestures are also effective in improving one's spatial thinking ability. Let's discuss each of these briefly.

One of the distinctive characteristics of human beings is that they can use symbolic representations, such as language, maps, diagrams, sketches, and graphs. Spatial language is a powerful tool for spatial learning. Babies learn a spatial relation better when it is given a name,<sup>18</sup> preschoolers who understand spatial words like "middle" perform better on spatial tasks than those who do not,<sup>19</sup> and preschool children whose parents use a greater number of spatial words (like *outside*, *inside*, *under*, *over*, *around*, and *corner*) show better growth in spatial thinking than children whose parents do not use such language.<sup>20</sup> Adults' spatial thinking is also enhanced by spatial language (e.g., the word *parallel* helps pick out an important spatial concept), as is their thinking about concepts, such as time, that are often described with spatial metaphors (e.g., *far in the future*).<sup>21</sup> Along similar lines, the ability to use maps can transform our thinking,<sup>22</sup> allowing us to draw conclusions that would be hard to arrive at without maps. A famous example is seeing the relation between drinking polluted water and getting cholera; in the 1800s, a map of water pumps in London superimposed on a map of cholera cases made the case for a relationship. Like maps, diagrams, sketches, and graphs also allow us to make inferences by supporting our spatial thinking.<sup>23</sup> For example, a graph of how boys and girls change in height over childhood and adolescence shows us very clearly that, on average, girls have an earlier growth spurt and finish growing earlier.

In addition to being able to think symbolically, humans have

\*Researchers are not sure why this is. It could be that those who are not good at spatial thinking have not yet developed mental strategies for dealing with spatial problems. So, in the initial stage when it appears that they are not improving, they could be developing and testing strategies. Then, once they have hit on an effective strategy, they start to improve and continue improving as they practice. In contrast, high-ability participants already have effective mental strategies and are simply becoming better through practice.

a distinctive ability to think analogically, that is, to see relational similarities between one situation and another. People can learn through noticing analogies, that is, by comparing two situations and noting their common relational structure (as when we compare the structure of the atom to the structure of the solar system). This process facilitates learning in children,<sup>24</sup> including spatial learning,<sup>25</sup> mathematical insight,<sup>26</sup> and scientific reasoning.<sup>27</sup> Thus, an additional way to get children to develop spatial reasoning abilities is to point out and highlight key comparisons they should be making.

People also gesture as they think, and gesture has turned out

Preschool children whose parents use a greater number of spatial words (like *outside*, *under*, *around*, and *corner*) show better growth in spatial thinking.



to be not only a window onto how thinking occurs,<sup>28</sup> but also a powerful tool for improving various kinds of learning. Gestures provide a window onto learners' minds and offer information about whether a learner is ready to improve on a task.<sup>29</sup> But gesture can also play a more active role in learning, in two ways. First, when *teachers* use gesture in instruction, children often learn better than when taught with speech alone.<sup>30</sup> Second, when *children* gesture as they explain a problem, either prior to<sup>31</sup> or during<sup>32</sup> instruction, they learn better than if they do not gesture. Gesture is a powerful means of reflecting and communicating about spatial knowledge.

Gesture has the potential to be a particularly powerful instructional tool in the spatial domain because it is particularly good at capturing spatial relations among objects. For example, when talking about how the earth turns and revolves around the sun, teachers can gesture to capture those relations.

Overall, our bag of tricks for enhancing spatial thinking is quite full. But there is more to learn. We know that practice, symbolic representations, analogies, and gestures all improve spatial thinking, but we don't know which of these approaches is most effective. Teachers will have to use their best judgment and fit spatial thinking into the school day as best they can. To help, I offer some suggestions at the end of this article.

### What about Sex Differences?

Sex differences are often the first thing people want to talk about when they consider spatial thinking. Three big questions usually come to mind: Do sex differences exist? If so, how big are they? What causes them—are they biological or environmental? Research has found sex differences in spatial thinking ability, both among average men and women, and among the very highest achievers. For some spatial tests, these differences are large. However, while these differences do exist, we need to remember that average sex differences do not tell us about individual perfor-

mance—some girls have strong spatial skills, and some boys are lacking these skills. Sex differences in spatial thinking are no barrier to women’s success in the STEM disciplines as long as educators take the steps to ensure that all students, of both sexes, acquire the spatial thinking skills they need.

The question about causes is a tricky one. The assumption behind this question is usually that, if biological, the difference is immutable, whereas if environmental, it could be reduced or even eradicated. There are two problems with the question, however. The first problem is with the assumption behind it: biological causation does *not* imply immutability, and environmental causation does *not* guarantee changeability. The second problem is that we don’t know the answer. A specially assembled team of experts with various takes on the problem recently concluded that there was evidence supporting both kinds of influences, with the additional possibility that the influences interacted (as when experience alters brain structures).<sup>33</sup>

Since spatial thinking can be improved, the important fact is not the causation of sex differences but the fact that girls (and boys) can improve. Some have suggested special training for females to help them catch up to males,<sup>34</sup> but as educators we want all students to do their best. That means we may not close the gap: meta-analyses have found that the sexes generally improve in parallel, and thus the sex difference continues even with training<sup>35</sup> (although some exceptions have been reported in which performance by men and women converged<sup>36</sup>). Nevertheless, even if the gap does not close, many women (and men) can and will come to perform well above threshold levels for success in the STEM disciplines, at which point other factors such as persistence, communication, and creativity may be more important than spatial ability.

## What Does This Mean for Teachers?

Since spatial cognition is malleable, spatial thinking can be fostered with the right kind of instruction and technology. As we have seen, spatial thinking improves during the school year more than over the summer months,<sup>37</sup> showing that teachers are helping students already. But what exactly should we be doing to help them improve even more? Unfortunately, precise answers are not yet possible. The National Academies’ report *Learning to Think Spatially* pointed out that we still lack specific knowledge of what kinds of experiences lead to improvement, how to infuse spatial thinking across the curriculum, or whether (and how best) to use new technologies such as Geographic Information Systems, especially with young children. What kinds of teaching best support spatial learning? Are these kinds of teaching different at different ages, at different socioeconomic status levels, or for girls and boys? Developing and testing curricula in a scientific way can be a slow process, and much remains to be done to be absolutely sure of our ground. However, we are beginning to have some good ideas about where to start, especially with preschool and elementary school students.

1. *Teachers (and parents) need to understand what spatial thinking is, and what kinds of pedagogical activities and materials support its development.* Recall that spatial thinking involves noticing and remembering the locations of objects and their shapes, and being able to mentally manipulate those shapes

and track their paths as they move. Because spatial thinking is not a subject, not something in which children are explicitly tested, it often gets lost among reading, mathematics, and all the other content and skills specified in state standards. Teachers need to be able to recognize where they can infuse it into the school day. For example, teachers could use the cardinal directions (north, south, east, and west) to talk about how to get to the cafeteria or playground, or use words like *parallel* and *perpendicular* when possible.

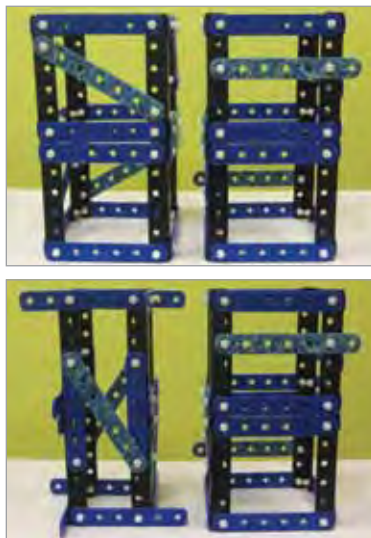
2. *Teachers at all levels need to avoid infusing students with anxiety about spatial tasks.* In general, anxiety about doing a task can impede performance, at least in part by occupying valuable mental space in working memory.<sup>38</sup> When you spend a lot of time worrying that you won’t do well, you lack the cognitive resources to actually concentrate on the work, a sad example of a self-fulfilling prophecy. Research with first- and second-graders in the Chicago Public Schools has recently shown that this vicious circle is evident for spatial thinking as well as for other areas like math: children who worry about not doing well perform more poorly than children who do not have such anxiety.<sup>39</sup> Thus, as is also true for other areas in teaching, teachers should avoid presenting spatial tasks as difficult challenges on which some people may not do well, or presenting students’ performance on these tasks as indicative of their underlying spatial abilities. Instead, teachers should emphasize that the tasks can be enjoyable and useful, and that they can be mastered with some effort and time.

3. *In the preschool years, teachers (and parents) need to encourage, support, and model engagement in age-appropriate spatial activities of a playful nature.* Preschool children need a good balance of play and formal instruction.<sup>40</sup> Fortunately, there is a wealth of spatial material available for preschool play, much of which can be further leveraged by a teacher with knowledge of the processes of spatial learning. Here are some specific ideas that could fit into most preschool settings:

- Select spatially challenging books for young children. For example, *Zoom* is a book in which attention continually zooms in to finer and finer levels of detail. Verbal and gestural support for children in dealing with the book’s conceptual and graphic challenges is correlated with children’s scores on spatial tests.<sup>41</sup>
- Use odd-looking as well as standard examples when teaching the names of geometric shapes such as circle, square, and triangle (e.g., a tipped, skinny, scalene triangle as well as an equilateral triangle pointing up). Showing these kinds of shapes supports learning that triangles are any closed figure formed by three intersecting straight lines.<sup>42</sup>
- Teach spatial words such as *out, in, outside, inside, middle, between, here, there, front, back, side, top, bottom, up, down, under, over, around, tall, high, short, low, line (it) up, row, next (to), and corner*. Learning spatial words can be enhanced by using gestures that highlight the spatial properties being discussed.<sup>43</sup>
- Encourage young children to gesture. Research has found

that when children are asked whether two shapes can be fit together to make another shape, they do significantly better when encouraged to move their hands to indicate the movements that would be made in pushing the shapes together.<sup>44</sup> Some children do this spontaneously, but children who do not will perform better when asked to gesture.

- Ask children to imagine where things will go in simple “experiments.” For example, preschoolers are prone to think that dropped objects will appear directly below where they were released, even when they are dropped into a twisting tube with an exit point far away. But, when asked to *visualize* the path before responding, they do much better. Simply being asked to wait before answering does not help—visualization is key.<sup>45</sup>
- Do jigsaw puzzles with children; they have been found to predict good spatial thinking, especially when coupled with spatial language (e.g., *Can you find all the pieces with a flat edge?*).<sup>46</sup> Similarly, play with blocks is a great activity in itself, and it increases use of spatial language.<sup>47</sup>
- Use maps and models of the world with children as young as 3.<sup>48</sup>
- Develop analogies to help young children learn scientific ideas, such as the principle of how a brace supports a building.<sup>49</sup> Consider the two photos below. In the one on top, comparing the two structures is relatively easy because the only difference is whether the brace is diagonal or horizontal, but on the bottom the comparison is more difficult because the two structures differ in several ways. When children shake these structures to see how much they wiggle, they are much more likely to conclude that a diagonal piece increases stability when interacting with the display on top.



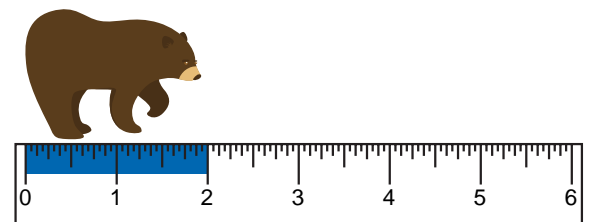
4. *In the elementary school years, teachers need to supplement the kinds of activities appropriate for preschoolers with more focused instruction in spatial thinking.* Playful learning of the sort that occurs in preschool can continue to some extent in

elementary school; activities such as block building, gesturing, reading spatially challenging books, etc., continue to develop spatial skills in older children too.<sup>50</sup> But as children get older, they can also benefit from more focused lessons. Mathematics is a central subject in which spatial thinking is needed, because space provides a concrete grounding for number ideas, as when we use a number line, use base-10 blocks, or represent multiplication as area. Here are some specific ideas for children in kindergarten through fifth grade:

- Highlight spatial elements in mathematics lessons. Measurement, for example, can be difficult for children to master, especially when the object to be measured is not aligned with the end of a ruler. Children often make mistakes such as counting hash marks beginning with 1, thus getting an answer that is one unit too many. When teaching measurement in the early grades, teachers can consider using a technique in which the unit between hash marks on a ruler is highlighted as the unit of measurement.<sup>51</sup> As shown in the illustration below, children can work with small unit markers coordinated with larger pieces to highlight how to determine units.



1. Measure the object so that it is not aligned with the beginning of the ruler. Place opaque unit pieces below the object to measure how long it is.



2. Move the object back to the beginning of the ruler, and use the unit pieces to “check” the answer.

- Add mapping skills, when possible, to geography lessons in the upper elementary grades. Some ideas can be found in Phil Gersmehl’s book, *Teaching Geography*, which is based in part on cognitive science.<sup>52</sup>
- Use well-crafted analogies so that comparisons will highlight essential similarities and differences. For example, students can compare diagrams of animal and plant cells to see similarities and differences.<sup>53</sup>
- Ask children in upper elementary and middle school to make sketches to elaborate on their understanding of top-

ics such as states of matter, or force and motion.<sup>54</sup> For example, they can be asked to draw water molecules in the form of ice, liquid, or vapor.

- Suggest beneficial recreational activities, such as photography lessons (to develop a sense of shifting viewpoints and changes in scale<sup>55</sup>), origami (to deepen their knowledge and skill in combining shapes) and JavaGami<sup>56</sup> (software for creating polyhedra), and video games like Tetris.<sup>57</sup>

**S**patial thinking is important, probably as important as verbal and mathematical thinking, for success in science, technology, engineering, and mathematics. Furthermore, it can be taught, and something we do in schools is already associated with improving it. Yet we can do better. The need to develop students' spatial thinking is currently not widely understood. We already have some excellent techniques for developing it, through practice, language, gesture, maps, diagrams, sketching, and analogy. Systematically building these techniques into the curriculum could yield important dividends for American education. □

## Endnotes

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**Answers to the sample test items on page 30: 1. A, 2. A, 3. C, 4. D.**